Değişkenlerin Ayrılabilir Dif.Denklemler

$y' = f(x,y) \Rightarrow A(x,y) + B(x,y)\dfrac{dy}{dx} = 0$

$A(x,y)\,dx + B(x,y)\,dy = 0$

$\int A(x)\,dx + \int B(y)\,dy = \int 0$

$F(x) + G(y) = C \longrightarrow$ genel çözüm

$\cancel{\text{ör}}$: $y' = \dfrac{6xy}{x^2+1}$ genel çözüm bulunuz.

$\dfrac{dy}{dx} = \dfrac{6xy}{x^2+1} \Rightarrow \dfrac{dy}{y} = \dfrac{6x\,dx}{x^2+1} \Rightarrow \int \dfrac{dy}{y} = 3\int \dfrac{2x\,dx}{x^2+1}$

$\Rightarrow \ln|y| = 3\ln|x^2+1| + \ln|c|$

$\ln|y| = \ln|(x^2+1)^3| + \ln|c|$

$\cancel{\ln}|y| = \cancel{\ln}|(x^2+1)^3 . c|$

$y = c.(x^2+1)^3$

$\cancel{\text{ör}}$ $2y\,dx - x\,dy = x.y\,dx$ genel çözüm bulunuz.

$x\,dy = 2y\,dx - xy\,dx \quad /xy \Rightarrow \dfrac{dy}{y} = \dfrac{2dx}{x} - dx$

$\Rightarrow \int \dfrac{dy}{y} = \int \left(\dfrac{2}{x} - 1\right)dx$

$\ln|y| = 2\ln|x| - x + \ln|c|$

$\ln|y| = \ln(x^2.c) - x$

$\ln|y/x^2c| = -x$

$\dfrac{y}{x^2.c} = e^{-x} \Rightarrow y = c.x^2.e^{-x}$

yö = ? Başlangıç değer

$y(1) = 2e \qquad \Rightarrow 2e = c.e^{-1}$

$\left.\Rightarrow 2e^2 = c \right\} \Rightarrow$ yö $= 2x^2.e^{2-x}$

Abdulrahman Hamdi tarafından yazıldı

Homojen dif. denklem sınıflandırması

$y' + p(x)\cdot y = Q(x)$ $\quad Q(x) = 0$

$\Rightarrow y' + p(x)\cdot y = 0$ Homojen

$a(x) y^{(n)} + b(x)\cdot y^{(n-1)} + \dots + y = 0$

$f\left(\frac{x}{y}\right), g\left(\frac{y}{x}\right)$

$\boxed{\begin{array}{l} \tan(y), \cos(y), \sin(y) \ \alpha \\ \ln(y), e^{y}, \sin(x), e^{x} \ \alpha \\ \text{sabit } \alpha \end{array}}$

Ör// $\frac{dy}{dx} = \frac{3y^2 - x^2}{2xy}$

$\Rightarrow$ 1. yol $\quad \frac{dy}{dx} = \frac{\frac{3y^2}{xy} - \frac{x^2}{xy}}{\frac{2xy}{xy}} = \frac{3\left(\frac{y}{x}\right) - \left(\frac{x}{y}\right)}{2}$ = Homojen dif. denklem

2. yol $\quad (x, y) \to (tx, ty)$

$\Rightarrow \frac{dy}{dx} = \frac{3t^2y^2 - t^2x^2}{2t^2xy} = \frac{\cancel{t^2}(3y^2 - x^2)}{\cancel{t^2}\cdot 2xy}$ 0. ıncı mertebeden dif. denklem

Ör// $(2x + y^2)\,dx - xy\,dy = 0$

$\frac{dy}{dx} = \frac{2}{y} + \frac{x}{y}$ varsa Homojen değil $\quad$ x veya y başına varsa Homojen değil

Ör// $y' + y = 3 \Rightarrow y = ?$

$y' + y = 0$ Homojen için

$\Rightarrow \frac{dy}{dx} + y = 0 \Rightarrow dy + y\,dx = 0$

$\frac{dy}{y} = -dx \Rightarrow \ln|y| = -x + \ln|c|$

$\frac{y}{c} = e^{-x} \Rightarrow y = c\cdot e^{-x}$

$y_h = c\cdot e^{-x}$ $\quad y_g = ?$, $y_ö = ?$

$y_h = c(x)\cdot e^{-x} \Rightarrow c'(x)\cdot e^{-x} - c(x)\cdot e^{-x} = y_h'$ $\quad y' + y = 3$

$\Rightarrow c'(x)\cdot e^{-x} - \cancel{c(x)\cdot e^{-x}} + \cancel{c(x)\cdot e^{-x}} = 3$

$c'(x)\cdot e^{-x} = 3$

$\int c'(x) = \int 3\cdot e^{x} \Rightarrow c(x) = 3\cdot e^{x} + c_1$

$y_g = \boxed{} \cdot \boxed{} \to$ Fonksiyon
değer
c(x).

$\Rightarrow y_g = (3\cdot e^{x} + c_1)\, e^{-x}$ ✓ $\quad 0 = 3 + c_1 \Rightarrow c_1 = -3$

$y(0) = 0$ $\quad y_ö = ? \Rightarrow y_ö = (3\cdot e^{x} - 3)\, e^{-x} \Rightarrow y_ö = 3 - 3\cdot e^{-x}$

c(x) değeri
başlangıç değer

Abdulrahman Hamdi tarafından yazıldı

Ör// $\frac{dy}{dx} - 3y = 7$ $y_ö = ?$ $y_g = ?$, $y_h = ?$

1. yol $y = e^{mx} \Rightarrow y' = m \cdot e^{mx}$ 2. mertebe yolu

$y' - 3y = 0 \Rightarrow m \cdot e^{mx} - 3e^{mx} = 0$

$\cancel{e^{mx}}(m-3) = 0$ $\Big\} \; y_h = C(x)_i \; e^{mx}$

sıfır vermez $\Rightarrow m = 3$ $\quad y_h = C(x) \cdot e^{3x}$

2. yol $\frac{dy}{dx} - 3y = 0$

$\frac{dy}{y} - 3dx = 0 \Rightarrow \frac{dy}{y} = 3dx$ $\quad y(0) = 0$ başlangıç değer

$\ln|y| = 3x + \ln|c|$

$\frac{y}{c} = e^{3x} \Rightarrow y = c \cdot e^{3x}$

$y_h = C(x) \cdot e^{3x}$

$y_g = ? \Rightarrow$ $y_h' = C'(x) \cdot e^{3x} + \cancel{3 \cdot e^{3x} C(x)} \cancel{- 3C(x) \cdot e^{3x}} = 7$

$C'(x) \cdot e^{3x} = 7$

$\int C'(x) = \int 7 \cdot e^{-3x} \, dx$

$C(x) = -\frac{7}{3} e^{-3x} + C_1$

$y_g = \left(-\frac{7}{3} e^{-3x} + C_1\right) e^{3x} \Rightarrow y_g = -\frac{7}{3} + C_1 \cdot e^{3x}$ $\Rightarrow 0 = -\frac{7}{3} + C_1$

$y_ö = -\frac{7}{3} + \frac{7}{3} \cdot e^{3x}$ $\quad C_1 = \frac{7}{3}$

$y_ö = \frac{7}{3}(e^{3x} - 1)$

Abdulrahman Hamdi tarafından yazıldı

Lineer (İntegral Çarpanı) Dif.Denklemler

$y' + p(x)\,y = Q(x) \quad \Rightarrow \mu(x) = e^{\int p(x)dx}$

2.adım $\quad (y\cdot\mu(x))' = Q(x)\cdot\mu(x)$

3.adım $\int \Rightarrow \quad y\cdot\mu(x) = \int Q(x)\cdot\mu(x)\,dx$

4.adımın $\quad y = \dfrac{\int Q(x)\cdot\mu(x)\cdot dx}{\mu(x)}$

Ör/ $2y' - y = 2e^{-x}$, $y(0)=1$ çözüm bulunuz

$y' - \frac{y}{2} = e^{-x} \qquad \mu(x) = e^{\int -\frac{1}{2}dx} \Rightarrow \mu(x) = e^{-\frac{x}{2}}$

$(y\cdot e^{-\frac{x}{2}})' = e^{-x}\cdot e^{-\frac{x}{2}}$

$\int (y\cdot e^{-\frac{x}{2}})' = \int e^{-\frac{3}{2}x}\,dx$

$y\cdot e^{-\frac{x}{2}} = -\frac{2}{3}e^{-\frac{3}{2}x} + C$

$y=1,\ x=0 \qquad 1\cdot e^{0} = -\frac{2}{3}\cdot e^{0} + C$

$\frac{2}{3} + 1 = C \Rightarrow C = \frac{5}{3}$

$\Rightarrow y = -\frac{2}{3}e^{-x} + \frac{5}{3}\cdot e^{\frac{x}{2}}$

Ör/ $t\cdot y' + 2y = 4t^2$, $y(1)=2$ çözüm bulunuz

$y' + \frac{2}{t}y = 4t \quad \Rightarrow p(t) = \frac{2}{t} \Rightarrow \mu(t) = e^{2\int\frac{1}{t}dt}$ $\qquad \boxed{e^{\ln x}} = x \qquad \cancel{\ln e} = x$ kural

$\mu(t) = e^{2\ln t}$

$\mu(t) = t^2$

$(y\cdot t^2)' = 4t\cdot t^2 \Rightarrow \int (y\cdot t^2)' = 4\int t^3\,dt$

$y\cdot t^2 = t^4 + C$

$y=2,\ t=1 \qquad y = t^2 + t^{-2}C$

$2 = 1 + C \Rightarrow C = 1$ $\qquad \frac{1}{t^2} + t^2$

$y = t^2 + t^{-2} \Rightarrow y = \dfrac{t^4+1}{t^2}$

Ör/ $y' + 2xy = x$ $\quad y(0)=0$ çözüm bulunuz

$p(x) = 2x \Rightarrow \mu(x) = e^{\int 2x\,dx} = e^{x^2}$ $\qquad e^{x^2} = u$

$(y\cdot e^{x^2})' = x\cdot e^{x^2} \Rightarrow y\cdot e^{x^2} = \int x\cdot e^{x^2}dx$ $\qquad \cancel{2}x\cdot e^{x^2}dx = \frac{du}{2}$

$y\cdot e^{x^2} = \frac{1}{2}\int du \Rightarrow y\cdot e^{x^2} = u + C$

$y\cdot e^{x^2} = e^{x^2} + C \Rightarrow y=0,\ x=0 \Rightarrow y = 1 - e^{-x^2}$

$C = -1$

Abdulrahman Hamdi tarafından yazıldı

Bernoulli Dif.Denklemler

$y' + p(x)y = Q(x).y^n$ dönüşüm yapılacak $\boxed{v = y^{1-n}}$

dönüşüm

Bernoulli → Lineer

ör/ $y' - \frac{2y}{x} = \frac{y^2}{x^2}$ çözüm bulunuz

$v = y^{1-n} \Rightarrow n = 2$ $\quad v = y^{1-2} \Rightarrow v = y^{-1} = \frac{1}{y}$

$v' = -y^{-2}.y'$

$-y^2.v' = y'$

$\Rightarrow -y^2.v' - \frac{2y}{x} = \frac{y^2}{x^2}$

$/-y^2 \Rightarrow v' + \frac{2}{x}\underbrace{\left(\frac{1}{y}\right)}_{v} = \frac{1}{x^2}$

$v' + \frac{2v}{x} = \frac{1}{x^2}$ Lineer dif. denklem

$p(x) = \frac{2}{x} \Rightarrow \mu(x) = e^{2\int \frac{1}{x}dx}$

$\mu(x) = e^{2\ln x} = x^2$

$\int (vx^2)' = \int \frac{x^2}{x^2} dx = \int dv$

$\Rightarrow v.x^2 = x + C \Rightarrow v = x^{-1} + C.x^{-2} \Rightarrow v = \frac{x+C}{x^2}$

$\frac{1}{y} = \frac{x+c}{x^2}$

$y = \frac{x^2}{x+C}$

ör/ $y' + y = y^2.e^x$ çözüm bulunuz

Bernoulli dif $\Rightarrow v = y^{1-n}$, $n = 2$

$\Rightarrow v = y^{-1} \Rightarrow v' = -y^{-2}.y'$

$v = \frac{1}{y}$ $\quad -y^2.v' = y'$

$\Rightarrow -y^2.v' + y = y^2.e^x$

$v' - v = -e^x$ Lineer dif

$\Rightarrow \mu(x) = e^{\int -dx} \Rightarrow \mu(x) = e^{-x}$

$\Rightarrow \int (v.e^{-x})' = \int -\cancel{e^{-x}}.\cancel{e^{x}} \Rightarrow v.e^{-x} = \int -dx$

$v.e^{-x} = -x + C$

$v = -x.e^x + C.e^x$

$\frac{1}{y} = -x.e^x + C.e^x$

$y = \frac{1}{C.e^x - x.e^x}$

Abdulrahman Hamdi tarafından yazıldı

Riccati Dif.Denklemler

$y' = p(x) \cdot y^2 + Q(x) \cdot y + R(x)$

$\boxed{\text{dönüşüm yapılacak } \quad y = v + y_1}$

$y_1(x)$ x'e bağlı

$v(x)$ x'e bağlı

$\text{Riccati} \xrightarrow{\text{dönüşüm}} \text{Bernoulli} \xrightarrow{\text{dönüşüm}} \text{Lineer}$ 3. adım vardır çözüm için

~~ör~~ $y' - y^2 = (1-2x)y + x^2 - x + 1$, $y_1(x) = x$ çözüm bulunuz.

$y' = y^2 + (1-2x)y + (x^2 - x + 1)$

$y' = p(x)y^2 + Q(x)y + R(x)$ Riccati dif denklem

$y = v + y_1(x)$

$y = v + x \Rightarrow y - x = v$

türev

$\Rightarrow y' = v' + 1$

$\Rightarrow v' + \cancel{1} = y^2 + (1-2x)y + x^2 - x + \cancel{1}$

$y^2 = (v+x)^2$

$(1-2x)(v+x)$

$v - 2xv + x - 2x^2$

$v' = v^2 + \cancel{x^2} + \cancel{2xv} + v \cancel{-2xv} + \cancel{x} \cancel{-2x^2} + \cancel{x^2} \cancel{-x}$

$v' - v = v^2$ Bernoulli dif denklem

$t = v^{1-n} \overset{n=2}{\Rightarrow} t = v^{-1}$

$t' = -v^{-2} \cdot v' \Rightarrow -v^2 \cdot t' = v'$

$-v^2 \cdot t' - v = v^2$

$t' + \frac{1}{v} = -1$

$t' + t = -1$ lineer dif denklem

$P(x) = 1 \qquad \mu(x) = e^{\int dx} \Rightarrow \mu(x) = e^x$

$\int (t \cdot e^x)' = \int -e^x$

$t \cdot e^x = -e^x + c$

$t = \frac{c}{e^x} - 1 \Rightarrow t = \frac{c - e^x}{e^x}$

$\frac{1}{v} = t \Rightarrow \frac{1}{v} = \frac{c - e^x}{e^x}$

$v = \frac{e^x}{c - e^x}$

$v = y - x \qquad \Rightarrow y - x = \frac{e^x}{c - e^x} \Rightarrow y = \frac{e^x}{c - e^x} + x$

Abdulrahman Hamdi tarafından yazıldı

Formundaki Dif.Denklemler

$y' + p(x) = Q(x) \cdot e^{ny}$

dönüşüm yapılacak $v = e^{-ny}$

örn/ $y' = x e^{-2y} + \frac{1}{x}$ çözüm bulunuz

$y' - \frac{1}{x} = x \cdot e^{-2y}$ formundaki dif. denklem

formundaki $\xrightarrow{\text{dönüşüm}}$ Lineer

$v = e^{-ny} \quad n = -2 \Rightarrow v = e^{2y}$

$v' = 2 \cdot e^{2y} \cdot y'$

$\frac{v'}{2 \cdot e^{2y}} = y'$

$\Rightarrow \frac{v'}{2 \cdot e^{2y}} - \frac{1}{x} = x \cdot e^{-2y}$

$v' - \frac{2(e^{2y})}{x} = 2x \cdot \cancel{e^{-2y} \cdot e^{2y}}$

$v' - \frac{2v}{x} = 2x$ Lineer dif denklem

$p(x) = -\frac{2}{x} \Rightarrow \mu(x) = e^{-2\int \frac{1}{x} dx} \Rightarrow \mu(x) = e^{-2\ln x}$

$\mu(x) = x^{-2} = \frac{1}{x^2}$

$\int \left(v \cdot \frac{1}{x^2}\right)' = \int 2 \cdot \frac{x}{x^2}$

$v \cdot \frac{1}{x^2} = 2 \cdot \int \frac{1}{x} dx \Rightarrow v \cdot \frac{1}{x^2} = 2 \ln x + C$

$v = 2x^2 \ln x + Cx^2$

$\ln e^{2y} = \ln 2x^2 \ln x + Cx^2$

$2y = \ln(2x^2 \ln x + Cx^2) \Rightarrow y = \frac{\ln(2x^2 \ln x + Cx^2)}{2}$

örn/ $y' + 3x = e^{3y} \cdot x$ formundaki dif. denklem çözünüz

$v = e^{-ny} \Rightarrow n = 3 \Rightarrow v = e^{-3y}$

$v' = -3 \cdot e^{-3y} \cdot y'$

$y' + 3x = e^{3y} \cdot x$

$\times / -3e^{-3y} \Rightarrow \underbrace{-3 \cdot e^{-3y} \cdot y'}_{} - 9 \cdot \underbrace{e^{-3y}}_{} \cdot x = -3 \cdot \cancel{e^{-3y} \cdot e^{3y}} \cdot x$

$v' - 9xv = -3x$ lineer dif denklem

$p(x) = -9x \Rightarrow \mu(x) = e^{-9\int x \, dx} \quad \mu(x) = e^{-\frac{9x^2}{2}}$

$\int \left(v \cdot e^{-\frac{9x^2}{2}}\right)' = \int -3x \cdot e^{-\frac{9x^2}{2}} \Rightarrow -\frac{9x^2}{2} = u \Rightarrow -3x \, dx = \frac{du}{3}$

$v \cdot e^{-\frac{9x^2}{2}} = \frac{1}{3} \int e^u \cdot du \Rightarrow \frac{1}{3} e^{-\frac{9x^2}{2}} + C$

$v \cdot e^{-\frac{9x^2}{2}} = \frac{e^{-\frac{9x^2}{2}}}{3} + C \Rightarrow v = \frac{1}{3} + C \cdot e^{\frac{9x^2}{2}}$

$\ln e^{-3y} = \ln \frac{1}{3} + C \cdot e^{\frac{9x^2}{2}}$

$-3y = \frac{\ln\left(C \cdot e^{\frac{9x^2}{2}} + \frac{1}{3}\right)}{-3}$

Abdulrahman Hamdi tarafından yazıldı

Tam Dif.Denklemler

$F(x,y)=C$

$$\frac{d}{dx}[F(x,y)] = \frac{d(C)}{dx}$$

$$\frac{d}{dx}[F(x,y)] = 0$$

$$\frac{d}{dx}[F(x,y)] = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\cdot\frac{dy}{dx} = 0$$

$$d[F(x,y)] = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = 0$$

önemli değil formül Açıklaması

$M(x,y)dx + N(x,y)dy = 0$ Tam dif denklem

Şart tam dif için $\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x}$

Ör// $(\cos x\cdot\cos y + 6x)\,dx - (\sin x\cdot\sin y + 6y)\,dy = 0$

$$\underbrace{(\cos x\cdot\cos y + 6x)}_{M(x,y)}dx + \underbrace{(-\sin x\cdot\sin y - 6y)}_{N(x,y)}dy = 0$$

$$\frac{\partial M(x,y)}{\partial y} = M_y = -\sin y\cdot\cos x$$

$$\frac{\partial N(x,y)}{\partial x} = N_x = -\sin y\cdot\cos x$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Tam dif denklem

Ör// $(2xy + 3x^2)dx + (2y^2 + x^2)dy = 0$

$$\Rightarrow \underbrace{(2xy + 3x^2)}_{M(x,y)}dx + \underbrace{(2y^2 + x^2)}_{N(x,y)}dy = 0$$

$$\frac{\partial M}{\partial y} = 2x$$

$$\frac{\partial N}{\partial x} = 2x$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Tam dif. denklem

Ör// $(3x^2 + 2y^2\cdot x)dx + (6y^2 + 3x^2y)dy = 0$

$$\underbrace{(3x^2 + 2y^2\cdot x)}_{M(x,y)}dx + \underbrace{(6y^2 + 3x^2y)}_{N(x,y)}dy = 0$$

$$\frac{\partial M}{\partial y} = 4xy$$

$$\frac{\partial N}{\partial x} = 6x\cdot y$$

$\frac{\partial N}{\partial x} \neq \frac{\partial M}{\partial y}$ Tam dif. değil

Tam dif. denklem çözümü

$M(x,y)dx + N(x,y)dy = 0$

1. yol $F(x,y) = \int M(x,y)\,dx + h(y) \Rightarrow d(F(x,y)) = N(x,y) = \underbrace{\square}_{}^{\text{y'ye bağlı türev}} + h'(y)$

2. yol $F(x,y) = \int N(x,y)\,dy + h(x) \Rightarrow d(F(x,y)) = M(x,y) = \square^{\text{x'e bağlı}} + h'(x)$

Ör/ $\frac{2xy}{1+x^2} - 2x - (2 - \ln(x^2+1))y' = 0$ çözüm bulunuz $\quad y' \to \frac{dy}{dx}$

$$\left(\frac{2xy}{1+x^2} - 2x\right) + (\ln(x^2+1) - 2)\frac{dy}{dx} = 0$$

$$\underbrace{\left(\frac{2xy}{1+x^2} - 2x\right)}_{M(x,y)}dx + \underbrace{(\ln(x^2+1) - 2)}_{N(x,y)}\,dy = 0$$

$$\left.\begin{aligned}\frac{\partial M}{\partial y} &= \frac{2x}{1+x^2}\\ \frac{\partial N}{\partial x} &= \frac{2x}{1+x^2}\end{aligned}\right\} \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{Tam dif. denklem}$$

$$F(x,y) = \int (\ln(x^2+1) - 2)\,dy + h(x)$$

$$F(x,y) = y\cdot\ln(x^2+1) - 2y + h(x) \quad \text{x'e bağlı Türev}$$

$* \; M(x,y) = \square + h'(x) \Rightarrow \cancel{\frac{2xy}{x^2+1}} + h'(x) = \cancel{\frac{2xy}{1+x^2}} - 2x$

$$\int h'(x) = \int -2x$$

$$h(x) = -x^2 + C$$

$$\Rightarrow y\cdot\ln(x^2+1) - 2y - x^2 = C \quad \text{genel çözüm}$$

2. yol $\quad F(x,y) = \int \left(\frac{2xy}{1+x^2} - 2x\right)dx + h(y)$

$$F(x,y) = y\cdot\ln(x^2+1) - x^2 + h(y)$$

$N(x,y) = \square + h'(y) \quad = \ln(x^2+1) + h'(y) \to$ Türev y'ye bağlı

$$\cancel{\ln(x^2+1)} + h'(y) = \cancel{\ln(x^2+1)} - 2$$

$$\int h'(y) = -2\int dy$$

$$h(y) = -2y + C$$

$$F(x,y) = C = y\cdot\ln(x^2+1) - x^2 - 2y \quad \text{genel çözüm}$$

Abdulrahman Hamdi tarafından yazıldı

Tam (Olmayan) hale getirebilir (İntegral Çarpanı) Dif.Denklemler

$M(x,y)dx + N(x,y)dy = 0$

$\Rightarrow \dfrac{\partial M(x,y)}{\partial y} = \dfrac{\partial N(x,y)}{\partial x}$ Tam dif. denklem

$\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$ Tam dif. değil (Tam olmayan)

* x'e bağlı $f(x) = \dfrac{M_y - N_x}{N(x,y)}$

integral çarpanı $\lambda(x) = e^{\int \frac{M_y - N_x}{N(x,y)} dx}$

$M(x,y)dx + N(x,y) = 0$ Tam dif değilse

$\Rightarrow \lambda(x) \cdot M(x,y)dx + \lambda(x) \cdot N(x,y) = 0$ Tam dif denklem

Ör// $\underbrace{(3x^2y + 2xy + y^3)}_{M(x,y)}dx + \underbrace{(x^2+y^2)}_{N(x,y)}dy = 0$

$M_y = \dfrac{\partial M}{\partial y} = 3x^2 + 2x + 3y^2$

$N_x \ \dfrac{\partial N}{\partial x} = 2x$

$\dfrac{\partial N}{\partial x} \neq \dfrac{\partial M}{\partial y}$ Tam dif değil

Çözüm $\dfrac{M_y - N_x}{N(x,y)} = \dfrac{3x^2 \cancel{+2x} + 3y^2 \cancel{-2x}}{x^2+y^2} = 3\left(\cancel{\dfrac{x^2+y^2}{x^2+y^2}}\right)$

$\lambda(x) = e^{\int \frac{M_y - N_x}{N(x,y)} dx} = e^{\int 3 dx} = e^{3x}$

$e^{3x}\underbrace{(3x^2y + 2xy + y^3)}_{M'(x,y)}dx + e^{3x}\underbrace{(x^2+y^2)}_{N'(x,y)}dy = 0$

$\dfrac{\partial M'}{\partial y} = e^{3x}(3x^2 + 2x + 3y^2)$

$\dfrac{\partial N'}{\partial x} = e^{3x}(3x^2 + 2x + 3y^2)$

$(e^{3x} \cdot x^2 + y^2 \cdot e^{3x})' = 3 \cdot e^{3x} x^2 + 2x \cdot e^{3x} + 3 \cdot e^{3x} y^2$

$e^{3x}(3x^2 + 2x + 3y^2)$

$\dfrac{\partial M'}{\partial y} = \dfrac{\partial N'}{\partial x}$ Tam dif denklem

$F(x,y) = \int e^{3x}(x^2+y^2)dy + h(x)$

$e^{3x}\left(x^2 y + \dfrac{y^3}{3}\right) + h(x) = F(x,y)$

$\cancel{3e^{3x} x^2 y} + \cancel{2xe^{3x}} \cdot y + h'(x) = \cancel{3x^2 y e^{3x}} + \cancel{2xy \cdot e^{3x}} + y^3 \cdot e^{3x}$

$\int h'(x) = \int y^3 \cdot e^{3x} \Rightarrow h(x) = \dfrac{y^3}{3} \cdot e^{3x} + C \Rightarrow C = e^{3x}\left(x^2 y + \dfrac{y^3}{3}\right) + \dfrac{y^3}{3} e^{3x}$

Abdulrahman Hamdi tarafından yazıldı

Tam hale getirebilir (İntegral Çarpan) Dif.Denklemler

y'ye bağlı $f(x) = \frac{N_x - M_y}{M(x,y)}$

integral çarpanı $\lambda(y) = e^{\int \frac{N_x - M_y}{M(x,y)} dy}$

ör// $\underbrace{(x+y)\sin y}_{M(x,y)}\, dx + \underbrace{(x\sin y + \cos y)}_{N(x,y)}\, dy = 0$

$\frac{\partial M}{\partial y} = x\cdot\cos y + \sin y + y\cos y$

$\frac{\partial N}{\partial x} = \sin y$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Tam dif değil

x'e bağlı $\Rightarrow \frac{M_y - N_x}{N(x,y)} = \frac{x.\cos y + \cancel{\sin y} + y\cos y - \cancel{\sin y}}{x.\sin y + \cos y} \cancel{=} \alpha$

y'e bağlı $\Rightarrow \frac{N_x - M_y}{M(x,y)} = \frac{\cancel{\sin y} - x.\cos y - \cancel{\sin y} - y\cos y}{1} = \frac{-\cos y(\cancel{x+y})}{\sin y\,(\cancel{x+y})}$

$= -\cot y$

$\lambda(y) = e^{-\int \cot y\, dy}$

$\lambda(y) = e^{-\ln(\sin y)} = (\sin y)^{-1} = \frac{1}{\sin y} = \csc y$

$\int \frac{\cos y}{\sin y} dy = \int \cot y\, dy$

$\sin y = u$

$\cos y\, dy = du$

$\Rightarrow \int \frac{1}{u} du = \ln|u| + c$

$= \ln(\sin y) + c$

$\underbrace{(\cancel{\csc y}\,(x+y).\cancel{\sin y})}_{M^*(x,y)} dx + \underbrace{(\csc y\,(x.\cancel{\sin y} + \cos y))}_{N^*(x,y)} dy = 0$

$\frac{\cos y}{\sin y} = \cot y$

$\underbrace{(x+y)}_{M^*(x,y)}dx + \underbrace{(x + \cot y)}_{N^*(x,y)}dy = 0$

$\frac{\partial M^*}{\partial y} = 1$

$\frac{\partial N^*}{\partial x} = 1$

$\frac{\partial M^*}{\partial y} = \frac{\partial N^*}{\partial x}$ Tam dif.denklem

$F(x,y) = \int (x+y)dx + h(y)$

$F(x,y) = \frac{x^2}{2} + yx + h(y) \overset{\text{Türev}}{\Rightarrow} \cancel{x} + h'(y) = \cancel{x} + \cot y$

$\Rightarrow \int h'(y) = \int \cot y \quad h(y) = \ln(\sin y) + c$

$\Rightarrow c = \frac{x^2}{2} + yx + \ln(\sin y)$ genel çözüm

Abdulrahman Hamdi tarafından yazıldı